AMERICAN UNIVERSITY OF BEIRUT
Math 218 – Linear Algebra and Applications

EXAM 1
Fall 2014

Name: _____ **ID:** _____

The backs of the pages may be used as scratch paper.
NO QUESTIONS ARE ALLOWED.
Time: 60 minutes

Circle your section number:

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2W	1W	11W	12F	4F	11F	2M	1M	11M	2M	3M	11R
1	2	3	4	5	6	7	8	9	10	11	12

Multiple choice					
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e

True or False		
10	T	F
11	T	F
12	T	F
13	T	F
14	T	F
15	T	F
16	T	F

Question	**Points**	**Score**
1	21	
2	10	
3	9	
4	10	
5	9	
6-9	20	
10-16	21	
Total	100	

Part I

Answer each of the following problems in the space provided for each problem.

1. (21 points) For which values of a and b does the system

$$\begin{cases} x + 3y - z = 3 \\ 2x + 5y - az = 0 \\ 3x + 7y + 2z = b \end{cases}$$

have a) no solution, b) a unique solution, c) infinitely many solutions?

2. (10 points) Let A be an invertible matrix such that $(A^{-1} + I)$ is invertible. Show that $(A + I)^{-1} = (A^{-1} + I)^{-1}A^{-1}$.

3. (9 points) Show that if $A^tA = A$ then A is symmetric and $A = A^2$.

4. (10 points) If A and B are 4×4 matrices with $|A| = 2$ and $|B| = 9$, find $|3AB^{-1}A^t|$.

5. (9 points) Let A be an $n \times n$ matrix. Show that if $(2A^2+3I)^2 = A+I$ then A is invertible.

Part II

Circle the correct answer to each of the following problems IN THE TABLE ON THE FRONT PAGE. Each correct answer is worth 5 points.

6. Let $A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$. If B is the inverse of A then the sum of the diagonal entries of B is

a) -1/3

b) 4/3

c) 1

d) -2

e) none of the above

7. If A and B are symmetric matrices then

a) AB is symmetric

b) $A + B$ is symmetric

c) AB is invertible

d) the diagonal entries of AB are all zero

e) none of the above

8. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 1$ then $\begin{vmatrix} 3a+2d & 3b+2e & 3c+2f \\ g & h & i \\ d+5g & e+5h & f+5i \end{vmatrix} =$

a) 3

b) -3

c) 15

d) -15

e) none of the above.

9. If $\begin{bmatrix} a & b & b & b \\ 0 & b & 1 & c \\ 0 & 0 & c & c \end{bmatrix}$ is in row echelon form then we must have

a) $a = b = c = 0$ or $(a = 1$ and $b = c = 0)$

b) $a \in \mathbb{R}$ and $b \neq 0$ and $c \in \mathbb{R}$

c) $a \neq 0$ and $b \in \mathbb{R}$ and $c \in \mathbb{R}$

d) $(a \neq 0$ and $b = c = 0)$ or $(a \neq 0$ and $b \neq 0$ and $c \in \mathbb{R})$

e) a, b, c are all nonzero

Part III

Determine whether each of the following statements is true or false. Circle the correct answer IN THE TABLE ON THE FRONT PAGE. Each correct answer is worth 3 points.

10. If $A^2 = A$ then $A = 0$ or $A = I$.

11. If A is not invertible then the system $Ax = b$ has infinitely many solutions.

12. If $A + B$ is invertible then A and B are invertible.

13. If y and z are solutions of the system $Ax = b$ then any linear combination of y and z is also a solution.

14. If the reduced row echelon form of the augmented matrix of a linear system has a row of zeros then the system has infinitely many solutions.

15. The vector $\begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$.

16. If $A = \begin{bmatrix} 3 & 5 & 7 & 0 \\ 1 & -4 & 9 & 1 \\ 9 & 15 & 21 & 0 \\ 1 & -2 & -3 & -4 \end{bmatrix}$ then A^t is not invertible.

SCRATCH PAGE —— DO NOT TEAR